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Method for Measurement of Coupling Constants*†

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The possibility of direct experimental determination of certain coupling constants or partial widths is examined. As an illustration of the proposed method the process $p \to N+\pi$ is discussed in some detail. Among other cases discussed are $\omega \to \rho+\pi$, $Y_0^* \to \bar{K}+N$ (to determine the Y_0^* spin), and $\Sigma \to \Lambda+\pi$.

A METHOD is proposed for measuring coupling constants, or partial widths of resonances which are small or normally inaccessible for kinematical reasons.[1] The idea is most readily pictured (with no loss of generality) in terms of observing breakup of a compound particle A into a particular set of its constitutent particles (let us say two particles 4 and 5). When breakup occurs exoergically, one directly observes the partial width, or, within a known factor, $g^2{}_{A45}$. We can, however, also observe the tail of a partial width to a channel that is normally inaccessible ($m_A < m_4+m_5$) by observing the breakup, at higher energy, of the virtually produced particle A. It is hoped that, although this virtual breakup cross section is small, it will dominate under appropriate conditions.

Consider the reactions

$$\sigma^{\mathrm{III}}:\ 1+2 \to 3+4+5\,, \tag{1}$$

$$\sigma^{\mathrm{II}}:\ 1+2 \to 3+A\,. \tag{2}$$

Let the total energy be W, four-momentum squared be Δ^2, and invariant energy of 4 and 5 be ω:

$$W^2=(p_1+p_2)^2,\quad \Delta^2=-(p_2-p_3)^2,\quad \omega^2=(p_4+p_5)^2.$$

If we assume that the dominant mode for (1) is the breakup (i.e., decay) of a resonance A [Fig. 1(a)], written σ_b, we can write

$$d\sigma^{\mathrm{III}}(\omega,W,\Delta)\approx d\sigma_b = \frac{\Gamma_{45}}{\pi}\frac{d\omega}{|\omega_r-\omega-i\Gamma|^2}N(\omega,W,\Delta)\frac{p_3'}{p_3}d\sigma^{\mathrm{II}}(W,\Delta)\,, \tag{3}$$

when we have integrated over the angles of the relative momentum of 4 and 5. $N(\omega)$ is an unknown function like a form factor for producing A off-the-mass shell, $N(\omega_r,W,\Delta)=1$. The factor p_3'/p_3 is a phase-space correction involving the momenta of the recoil in the over-all center of mass. We are interested in determining the partial half-width Γ_{45}. The standard procedure is to measure the total width from the shape of $d\sigma^{\mathrm{III}}(\omega)$ and to measure the branching ratios in the various channels. We propose here that when this simple method is not adequate (i.e., $m_A < m_4+m_5$, or the orbital angular momentum in the system 4+5 is large and $m_A \approx m_4+m_5$), Γ_{45} can still be measured by comparing the cross section (1) to (2) at the same W and Δ but at higher energy ω such that breakup into 4 and 5 is probable.

The expression analogous to (3) is readily obtained for the case where A is a bound or stable state (i.e., using perturbation theory):

$$d\sigma_b=\frac{G^2}{\pi}\frac{d\omega}{(\omega^2-M^2)^2}k^{2l+1}N\frac{p_3'}{p_3}d\sigma^{\mathrm{II}}\,, \tag{4}$$

where k is the relative momentum of particles 4 and 5 in their center of mass and G is the coupling constant g_{A45}. Here we have neglected final-state scattering, and breakup into other channels. To remove this restriction,

$$M-\omega \to \omega_r-\omega-i\Gamma\,, \tag{5}$$

with $k\to i\kappa$, $\kappa>0$, below threshold in any channel. A good relation between partial half-width and coupling constant is

$$\Gamma_{45}=\frac{G^2}{4m_A{}^2}\left[\frac{k^2}{1+r^2(k^2-k_0{}^2)}\right]^l k\,, \tag{6}$$

with k_0 the value of k at the resonance or bound state, and r an appropriate radius. Using (5) and (6), (4) is essentially the same as (3).

The production (1) may proceed by other modes. We argue that at high total energy W, other modes will spread all over the three-body phase space in ω, while the breakup process is concentrated at ω near m_A because of the pole. Thus, at fixed ω, other modes become smaller compared to $d\sigma^{\mathrm{II}}$, as W is increased, while $d\sigma_b$ remains fixed. (Note that the total σ^{III} and

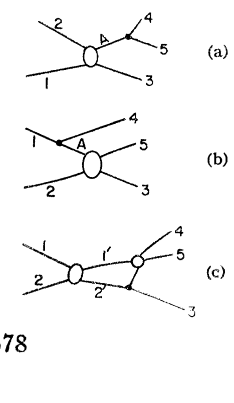

FIG. 1. The breakup process (a), and related processes (see text).

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† A preliminary version of this work is contained in the Proceedings of the Athens Conference on Recently Discovered Resonant Particles, Ohio University, 1963 (unpublished).
[1] Similar possibilities are considered in M. L. Good and W. D. Walker, Phys. Rev. **120**, 1857 (1960), and P. T. Matthews and A. Salam, Nuovo Cimento **12**, 126 (1961).

$\sigma^{\rm II}$ do not increase with phase space at high energy, but must slowly decrease with energy if the total cross section is to remain constant.) A rough criterion for sufficiently high W is that the fraction of three-body phase space appropriate to observe the break-up process (this would correspond, for example, to $\Delta\omega\approx m_4+m_5-m_A$, or perhaps to $\Delta\omega\approx$ smaller of m_4, m_5, etc.) should be small. For example, for $p-p$ scattering at 13 BeV in the lab where we observe the breakup $p\rightarrow N+\pi$ (see below), the three-body phase space ratio will be

$$\frac{\Delta\rho_3}{\rho_3}=8W^2\int\frac{kd\omega}{(W^2-4M^2)^2},$$

which will be about 3% if we observe the breakup in the region $M+\mu<\omega<M+3\mu$, and about 10% at 5 BeV. Thus, it seems practical to choose W large enough so that the breakup process is likely to dominate for small ω. There are two additional handles on the breakup process: (1) It is proportional to $d\sigma^{\rm II}(W,\Delta)$ which will probably be a rapidly varying function of Δ. (2) An extrapolation to $\omega=m_A$ is, in principle, possible. If the data permit an extrapolation, the coupling constant is determined as the residue at the pole.[2] Further aspects of these points are discussed below. The disadvantages of the proposed method are: (1) The breakup cross section will be small. (2) When the breakup system is in a p state (as seems to be typical), there is a strong zero (k^3) in the cross section at threshold, i.e., between the data and the pole in question.

To clarify the proposal, we consider as an exercise the measurement of $g_{NN\pi}$ by comparing

$$p+p\rightarrow p+(N+\pi) \tag{7}$$

and

$$p+p\rightarrow p+p. \tag{8}$$

Experiments at high W have been done by Cocconi *et al.*[3] They observed the fast recoil proton in the lab.

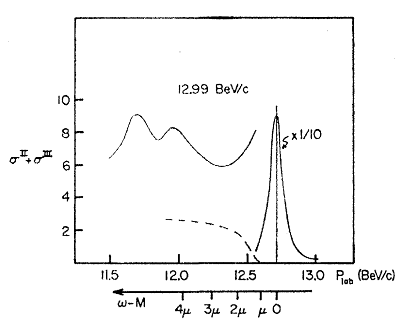

FIG. 2. The cross section for reaction (7) at 12.99 BeV/c as a function of recoil proton momentum (or ω) taken from Ref. 2. The dashed curve represents theory for the $p\rightarrow N+\pi$ breakup process (see text).

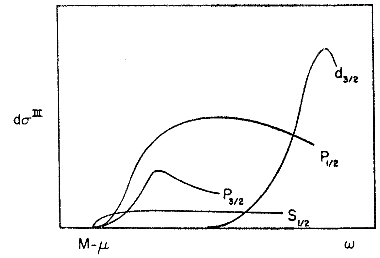

FIG. 3. Sketch suggesting the form of partial wave contributions to $d\sigma^{\rm III}(\omega)$ for reaction (7).

Note that at high W the recoil and breakup protons are completely distinguished by being fast or slow in the lab; interference between them being negligible. Calculating (7) in terms of (8) using pseudovector coupling, one obtains (4)[4] and finds that $G^2\mu^2/4M^2$ is the usual PV coupling constant

$$G^2\mu^2/4M^2\approx 0.09. \tag{9}$$

Of course, perturbation theory reveals the correct pole and threshold behavior shown in (4), but the prediction for $N(\omega)$ is worthless. In the numerical work below, we take $N=1$. It turns out that the Cocconi experiment does not have quite enough resolution for our purposes (of course, the experiment was not designed for these purposes); the resolution can, in principle, be easily improved by observing the breakup system. In Fig. 2, calculated results using (4) and (5) and the measured elastic cross section $d\sigma^{\rm II}$ shown in the figure, are shown as the dashed curve. It is seen that an upper bound on $g^2{}_{NN\pi}$ roughly twice its actual value is already set by this experiment. It is of interest to note that in the interval $M+\mu<\omega<M+3\mu$ the calculated value of $\int d\sigma^{\rm III}/d\sigma^{\rm II}$ is 4%.

In terms of the angular momentum states in the center-of-mass system of 4+5 ($N+\pi$), we can write $d\sigma^{\rm III}$ as a sum of partial cross sections (Fig. 3). The experiment would become convincing if one could determine the shape of $d\sigma^{\rm III}(\omega)$ well enough to distinguish the S-wave contribution and to say something about the $p_{3/2}$ and $d_{3/2}$ nucleon isobar contributions (the latter is already clearly seen in the Cocconi experiment).

We return to the general problem to discuss two miscellaneous points. At small momentum transfer the process of Fig. 1(b) will contribute and be concentrated at small ω. This process will have completely different momentum transfer behavior than the breakup process and lacks the pole at m_A, having a more distant cut instead. We can discriminate against it by going to larger momentum transfer (e.g., $\Delta^2\gtrsim M^2/2$ in the $p-p$ example discussed above). A second process, Fig. 1(c), is, in part, already included in σ_b. This is the process which might most obviously contribute anomalous singularities. It has been studied for its analytic

[2] The analogy of many aspects of this proposal to the Chew-Low [G. F. Chew and F. E. Low, Phys. Rev. **113**, 1640 (1959)] proposal for measuring inaccessible cross sections is obvious [e.g., point (2) just below]. It is hoped that as a result of point (1) above and the known dependencies illustrated by Fig. 3 below, the validity of the present method will be subject to convincing experimental tests.

[3] G. Cocconi, A. N. Diddens, E. Lillethun, G. Manning, A. E. Taylor, T. G. Walker, and A. M. Wetherell, Phys. Rev. Letters **7**, 450 (1961).

[4] There is also a small, $O(\mu^4/M^2)$ compared to k^2, s-wave ($\pi+N$) production term associated with the antinucleon intermediate state.

properties in ω by Landshoff and Treiman.[5] They find that at high W ($W\gg m_1'+m_2'$), there will be no singularity in ω near the normal thresholds, so that an extrapolation from $\omega=m_4+m_5$ to m_A is, in principle, possible.

Let us consider some possible applications for the method. Since the function $N(\omega)$ of (3) or (4) is not known and we look to the pole in σ_b to help it dominate other production mechanisms, we must do experiments as near the pole $\omega=m_A$ as possible. With this in mind, we list several experiments with recoil and breakup breakup particles first and second, respectively, on the right-hand sides of the reactions:

$$\pi^-+p\to n+\omega \qquad (10\text{-II})$$
$$\to n+\rho^\pm+\pi^\pm \qquad (10\text{-III})$$
$$\to n+K+\bar{K}\,,$$

$$\pi^-+p\to p+\rho^- \qquad (11\text{-II})$$
$$\to p+\omega+\pi^- \qquad (11\text{-III})$$
$$\to p+K^0+K^-\,,$$

$$K^-+p\to p+K^{*-} \qquad (12\text{-II})$$
$$\to p+K^-+\eta\,, \qquad (12\text{-III})$$
$$p+p\to p+(N^{**})^+ \qquad (13\text{-II})$$
$$\to p+p+\eta$$
$$\to p+\Lambda+K^+ \qquad (13\text{-III})$$
$$\to p+p+\rho^0$$
$$\to p+p+\omega\,,$$

$$K^-+p\to\pi^++\Sigma^- \quad (\text{or } \pi^-+\Sigma^+)\,, \qquad (14\text{-II})$$
$$(\text{or } \pi^-+p\to K^++\Sigma^-)$$
$$\to\pi^++\Lambda+\pi^-\,, \qquad (14\text{-III})$$
$$K^-+p\to\pi^-+(Y_1^*)^+ \qquad (15\text{-II})$$
$$\to\pi^-+K^0+p\,, \qquad (15\text{-III})$$

$$\pi^-+p\to K^0+Y_0^*(1405\text{ MeV}) \qquad (16\text{-II})$$
$$\to K^0+\bar{K}+N\,. \qquad (16\text{-III})$$

Also one could consider the analogous case:

$$\pi^-+p\to n+\eta$$
$$\to n+\pi^++\pi^-+\pi^++\pi^-\,. \qquad (17)$$

The experimental difficulties are, in many cases, enormous. Of course, in cases like (10) and (17) one may use π^++d.

A few remarks can be made about specific cases. In (10) it may be assumed that the on-the-mass shell ω already decays into $\rho+\pi$ (with ρ far below *its* resonant energy), while a high-mass ω can decay into $\rho+\pi$ with the ρ on the mass shell. To investigate this crudely, we estimate the dependence of the ω half-width on energy from

$$\Gamma\sim\int d\epsilon\frac{\Gamma_\rho(\epsilon)}{(m_\rho-\epsilon)^2+\Gamma_\rho{}^2}\frac{q^3}{1+r^2q^2}\,,$$

TABLE I. Number of events in various intervals of ω mass.

Energy of the ω decay (MeV)	Relative number of ω's
760–800	1.0
800–900	0.09
900–1000	0.08
1000–1100	0.06
1100–1200	0.05

where ϵ is the energy of two pions in their center-of-mass system and q the momentum of the third in the over-all center-of-mass system, and the half-width $\Gamma_\rho=\alpha k^3/(1+r^2k^2)=50$ MeV at resonance. The ω-decay distribution is found with this width to have a long, low tail on the high side. The number of events in various intervals of ω mass is shown in Table I for the case of ω full width at the peak of 5 MeV (this is near the optimum width for the effect). There is a broad weak secondary maximum in the distribution near 1 BeV which does not explicitly appear in the table. If the ω width were energy-independent, the relative number of events above 800 MeV in Table I would be 0.044. If a somewhat larger width at the ω peak were chosen, the number of events above 900 MeV would decrease. A smaller width would shift the peak in the tail to higher energy, the number of decays in a given interval would decrease.

In cases (11) and (14) the two-body cross sections are large and the interpretation relatively clean cut, in (14) the pole being close. The couplings involved, $g(\Sigma\Lambda\pi)$ and $g(\rho\omega\pi)$, are of great theoretical interest.

Case (16) is the only one which may involve s-wave breakup. The question of interest is not at present the coupling constant, but the spin and parity of Y_0^*. The $\frac{1}{2}^-$ hypothesis could be tested by looking for s-wave $T=0$ $\bar{K}N$ production at low energy, proportional to Y_0^* production. As always one has to find a situation where Y_0^* is produced cleanly, presumably at high energy. Adopting the $\frac{1}{2}^-$ hypothesis and using the approximation $\mathrm{Re}(1/a)=\text{const}$, where a is the $\bar{K}N$ $T=0$ scattering length in Humphrey-Ross solution 2,[6] the $\Sigma\pi$ peak occurs near the correct energy (1405 MeV) and one finds the Y_0^* decay ratio

$$\bar{K}N/\Sigma\pi=\tfrac{1}{3}\,,$$

where the denominator is integrated around the Y_0^* peak from 1375 to 1435 MeV and the numerator from threshold (1435) to 1535 MeV.

In several cases, (13), (14), and (15), one has to worry about breakup processes from other nearby poles, as in the $p\to N+\pi$ case above. One again hopes that the distributions of the two-body processes $d\sigma^{\mathrm{II}}(\Delta)$, and the threshold dependences of the partial contributions to $d\sigma^{\mathrm{III}}(\omega)$ will differ sufficiently to allow distinction between the various pole terms. One would

[5] P. Landshoff and S. Trieman, Phys. Rev. **127**, 649 (1962).

[6] W. E. Humphrey and R. R. Ross, Phys. Rev. **127**, 1305 (1962).

like to suggest looking at $(\bar{K}+N)$ as a breakup of Σ in (14), but in view of the Y_1^* and Y_1^{**} contributions this would be particularly difficult. We also note that the experiments on boson systems (10), (11), and (12) necessarily involve looking at breakup systems which are fast in the lab and, thus, more difficult to measure. In view of these difficulties, it may be guessed that most of these experiments will require highly refined equipment triggered on the processes in question. But it is just possible that some could be performed as a byproduct of large bubble-chamber experiments (note 4% estimate in $p\rightarrow N+\pi$ case above). As another example, reaction (14) could be examined at say 5 BeV: Consider only events associated with one very fast π in the lab. If several hundred slow charged Σ's were found, determining $d\sigma^{II}(\Delta)$, one could plot individual $(\Lambda+\pi)$ events, in a suitable ω interval, as a function of Δ weighted by $1/d\sigma^{II}(\Delta)$. There might be several tens of such events. The distribution should be constant. The weighted $\Lambda+\pi$ events for all suitable Δ should also be plotted against ω to test the k^3 dependence. The charged Y_1^*, distribution should also be examined to show, with luck, that the low ω $(\Lambda+\pi)$ events of interest were not the tail of that distribution. Passing all these tests, $d\sigma^{III}/d\sigma^{II}$ would be interpretable by means of (4) in terms of $g^2{}_{\Lambda\Sigma\pi}$.

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Y_0^* and the Low-Energy $\bar{K}N$ Interaction

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An analysis is attempted on the low-energy $\bar{K}N$ data by assuming that the Y_0^* is an S-wave $\bar{K}N$ bound state. Two tentative sets for the scattering lengths are obtained which are similar to that of Akiba and Capps, and fit all the low-energy data reasonably well.

The two solutions obtained by Humphrey and Ross[1] for the $\bar{K}N$ scattering lengths suggest the existence of a Dalitz-Tuan type resonance[2] in the $I=0$ state. It is, however, not yet conclusive quantitatively whether it corresponds to the Y_0^* recently observed.[3,4] There is some indication that the imaginary part of the isosinglet scattering length is too large for the above identification to be valid, especially in the first solution (hereafter referred to as HR-I).[5–8] Also it seems rather difficult to obtain a unique solution from the data on low-energy K^-p reactions only.[9] In this note we shall make an analysis on the low-energy $\bar{K}N$ data assuming that the Y_0^* has a spin $\frac{1}{2}$ and even parity with respect to KN.

In the zero-range approximation the (real) phase shift δ of isosinglet $\pi\Sigma$ scattering below the $\bar{K}N$ threshold is given by[2,8,10]

$$\frac{q}{\bar{q}}\cot\delta(\kappa)\approx-\frac{1}{z}\frac{\kappa_0}{\kappa_r}\frac{\kappa-\kappa_r}{\kappa-\kappa_0}, \tag{1}$$

where q is the $\pi\Sigma$ momentum, $\bar{q}$ being its value at $\bar{K}N$ threshold; κ may be taken as the average of the absolute value of the (imaginary) K^-p and $\bar{K}^0n$ momenta, and

$$\kappa_r=-(a_0+b_0z)^{-1}, \tag{2a}$$

$$\kappa_0=-(a_0-b_0/z)^{-1}, \tag{2b}$$

where $A_0=a_0+ib_0$ is the isosinglet $\bar{K}N$ scattering length, z being given by

$$z=\tan\varphi,$$

with φ, the value of δ at $\bar{K}N$ threshold. At κ_r and κ_0 there occur a peak and a dip, respectively, in the $\pi\Sigma$ scattering cross section.

The pole at κ_0 in the inverse of the $\pi\Sigma$ scattering amplitude is peculiar to a two-channel problem; an example of such a pole can be easily found in a simple chain approximation model.[10] The location of κ_0 varies

[1] W. E. Humphrey and R. R. Ross, Phys. Rev. **127**, 1305 (1962).
[2] R. H. Dalitz and S. F. Tuan, Ann. Phys. (N. Y.) **10**, 307 (1960).
[3] M. H. Alston, L. W. Alvarez, P. Eberhard, M. L. Good, W. Graziano, H. K. Ticho, and S. G. Wojcicki, Phys. Rev. Letters **6**, 698 (1961).
[4] G. Alexander, G. R. Kalbfleisch, D. H. Miller, and G. A. Smith, Phys. Rev. Letters **8**, 447 (1962).
[5] Y. Miyamoto, Progr. Theoret. Phys. (Kyoto) **27**, 203 (1962).
[6] T. Akiba and R. H. Capps, Phys. Rev. Letters **8**, 457 (1962).
[7] T. Akiba, Progr. Theoret. Phys. (Kyoto) **29**, 439 (1962).
[8] Y. Fujii, Progr. Theoret. Phys. (Kyoto) (to be published).
[9] For example, the observable results calculated from Humphrey and Ross's two solutions are distinguished with each other mainly by a delicate energy dependence of the ratio Σ^-/Σ^+.
[10] Y. Fujii and M. Uehara, Suppl. Progr. Theoret. Phys. (Kyoto) **21**, 138 (1962).